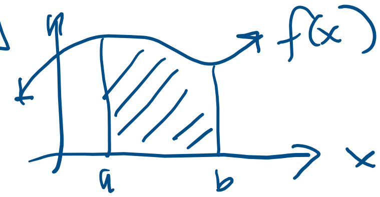
Recall: Definite Integral

If f is defined on $[a,b]$ then $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n} f(x_i)\Delta x$ where $\Delta x = \dfrac{b-a}{n}$ and $x_i = a + \Delta x \cdot i$.

has bounds $= \boxed{\int_a^b f(x)\,dx}$



First, practice evaluating some indefinite integrals: no bounds $\therefore$ add $+C$

Do: $\displaystyle\int \frac{\sqrt[5]{x^3}}{11}\,dx$

$$= \int \frac{x^{3/5}}{11}\,dx = \frac{1}{11}\int x^{3/5}\,dx$$

$$= \frac{1}{11}\,\frac{x^{8/5}}{8/5} + C = \frac{1}{11}\cdot\frac{5}{8}x^{8/5} + C$$

$$= \boxed{\frac{5x^{8/5}}{88} + C}$$

$$\int f(x)\,dx = \int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

Do: $\displaystyle\int \frac{2}{x}\,dx = 2\int \frac{1}{x}\,dx$

$(\ln x)' = \dfrac{1}{x}$ $\quad = \boxed{2\ln|x| + C}$

Do: $\displaystyle\int \sec x \tan x\,dx = \boxed{\sec x + C}$

$(\sec x)' = \sec x \tan x$

Evaluating Indefinite Integral with an Initial Condition

ex. $\int 8\, dx$ where $F(1)=4$.

$\leftarrow f(x)$ x y

$F(x) = 8x + C$

$F(1) = 8(1) + C = 4$

$C = -4$

$\boxed{F(x) = 8x - 4}$

$(8x)' = 8$

"Reverse" Power Rule

$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$

ex. $\int (3x^2 - 4x + 7)\, dx$ where $F(0)=8$.

$= \cancel{3}\frac{x^3}{\cancel{3}} - \cancel{4}^{2}\frac{x^2}{\cancel{2}} + 7x + C$

$f(x) = x^3 - 2x^2 + 7x + C$

$F(0) = 0 - 0 + 0 + C = 8$

$\therefore C = 8$

$\boxed{F(x) = x^3 - 2x^2 + 7x + 8}$

ex. $\int (\sin x - e^x)\, dx$ where $F(0)=9$.

$= \int \sin x\, dx - \int e^x\, dx$

$f(x) = -\cos x - e^x + C$

$F(0) = -\cos 0 - e^0 + C = 9$

$-1 - 1 + C = 9$

$-2 + C = 9$

$C = 11$

$\boxed{F(x) = -\cos x - e^x + 11}$

$(\cos x)' = -\sin x$

$(-\cos x)' = \sin x$

$(e^x)' = e^x$

A few more indefinite integrals that require manipulation:

FOIL

ex. $\int (3x-2)(2x+7)\ dx$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$= \int (6x^2 + 17x - 14)\,dx$$

$$= \frac{\cancel{6}^{2} x^3}{\cancel{3}} + \frac{17x^2}{2} - 14x + C$$

$$= \boxed{2x^3 + \frac{17}{2}x^2 - 14x + C}$$

$$\frac{1}{2} + 1 = \frac{3}{2}$$

ex. $\int \sqrt{x}(x+3)\ dx$ $= \int x^{1/2}(x^1 + 3)\,dx$

$$\frac{3}{2} + \frac{2}{2} = \frac{5}{2}$$

$$= \int \left(x^{3/2} + 3x^{1/2}\right) dx$$

$$= \frac{2}{5}x^{5/2} + \cancel{3}\cdot\frac{2}{\cancel{3}}x^{3/2} + C$$

$$= \boxed{\frac{2}{5}x^{5/2} + 2x^{3/2} + C}$$

$$\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$

ex. $\int \frac{4x^3 - 7x^2}{x}\ dx$ $= \int \frac{4x^{3}}{x}\,dx - \int \frac{7x^{2}}{x}\,dx$

$$= 4\int x^2\,dx - 7\int x^1\,dx$$

$$= \boxed{\frac{4}{3}x^3 - \frac{7}{2}x^2 + C}$$

Evaluating a Definite Integral

no +C

FTC Part 2: $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b)-F(a)$ where $F'=f$ (F is antideriv. of f)

assuming f is continuous on $[a,b]$

ex. $\int_1^3 5x\,dx$
$= 5\int_1^3 x\,dx$
$= \frac{5}{2}x^2\Big|_{a=1}^{b=3}$
$F(b) - F(a)$
$= \frac{5}{2}(3^2 - 1^2)$
$= \frac{5}{2}(9-1) = \frac{5}{\cancel{2}}\cdot \cancel{8}^{4} = \boxed{20}$

ex. $\int_1^e \frac{3}{x}\,dx$
$= 3\int_1^e \frac{1}{x}\,dx$
$= 3\cdot \ln|x|\Big|_1^e$
$= 3(\ln e - \ln 1)$
$= 3(1-0) = \boxed{3}$

$(\ln x)' = \frac{1}{x}$

$\ln|e| = \ln e = 1$
$\ln|1| = \ln 1 = 0$

ex. $\int_0^1 (e^x + x^2 - 9)\,dx$
$= \left(e^x + \frac{x^3}{3} - 9x\right)\Big|_0^1$
$= e^x\Big|_0^1 + \frac{1}{3}x^3\Big|_0^1 - 9x\Big|_0^1$
$= e^1 - e^0 + \frac{1}{3}(1-0) - 9(1-0)$
$= e - 1 + \frac{1}{3} - 9$
$= e - 10\cdot\frac{3}{3} + \frac{1}{3} = \boxed{e - \frac{29}{3}}$

$\frac{x^3}{3}\Big|_0^1 = \frac{1}{3}\left(x^3\Big|_0^1\right)$
$= \frac{1}{3}(1^3 - 0^3)$
$= \frac{1}{3}(1-0)$

ex. $\int_1^4 7\sqrt{x}\,dx$
$= 7\int_1^4 x^{1/2}\,dx$
$= 7\cdot\frac{2}{3}x^{3/2}\Big|_1^4$
$= \frac{14}{3}\left(4^{3/2} - 1^{3/2}\right)$
$= \frac{14}{3}(8-1) = \frac{14}{3}\cdot 7 = \boxed{\frac{98}{3}}$

$\frac{1}{2}+1 = \frac{3}{2}$

$4^{3/2} = (4^{1/2})^3 = \sqrt{4}^3$
$= 2^3 = 8$

$\begin{array}{r} {}^{2}14 \\ \times 7 \\ \hline 98 \end{array}$

Fundamental Theorem of Calculus

Now, on to examples of FTC Part 1:

inverses

derivatives $\Longleftrightarrow$ integrals

Recall:

FTC Part 1: $\int_a^x f(t)\,dt = g(x)$ where f is continuous on $[a,b]$

a is a constant

lower case

FTC Part 1: (rewritten) $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)\cdot x'$

integral

Use FTC Part 1 :)

ex. Find the derivative of $\int_0^x \sqrt{1+t^2}\,dt = \sqrt{1+x^2}\cdot \underset{1}{x'} = \boxed{\sqrt{1+x^2}}$

no +C

check lower bound is a constant

then plug in upper bound into function's t

ex. Determine $\frac{d}{dx}\int_0^x 7t^3\,dt$ $= 7x^3 \cdot \underset{x'}{1}$

$= \boxed{7x^3}$

ex. Use Chain Rule to determine $\frac{d}{dx}\int_1^{x^4} \ln t\,dt$ $= \ln x^4 \cdot (x^4)'$

$= \ln x^4 \cdot 4x^3$

$= \boxed{4x^3 \cdot \ln x^4}$